**Calculus AP Summer Packet**

**2011**

**Mrs. DeWitt**

The problems included in this packet represent the material normally covered in a college pre-calculus course. The topics covered include material needed for the first two calculus courses offered at the college level. The assumption when taking calculus at any level is that every student has been exposed to these topics and, if a review is necessary, the student will review on their own. With this in mind, calculus concepts are covered the first day in a calculus class.

I do not make that assumption. I deal with the reality that even if all these topics were covered previous to calculus, a review is necessary. Even though the first three weeks of school are devoted to a review of these topics, three weeks of 45 minute classes will not be sufficient time to go over all of these problems. Students must find time over the summer to do as much as they can in this packet. You will be tested on this packet starting the first week of classes.

**Would you like to be an AP Calculus SURVIVOR? With these skills as lifelines, you are sure to make a "5."**

Factor out monomial factors as indicated:

1. $3x^3 + 4x^3 - x^2 = x^2($ $)$
2. $\frac{x}{2} - 6x^2 = \frac{x}{2}($ $)$
3. $2\sqrt{x} + 6x^{3/2} = 2\sqrt{x}\left($ $\right)$
4. $\sin x + \tan x = \sin x($ $)$
5. $e^{-x} - xe^{-x} + 2xe^{-x} = e^{-x}($ $)$
6. $x^{-1} - 2 + x = x^{-1}($ $)$
7. $\frac{1}{2x^2 + 4x} = \frac{1}{2x}($ $)$

Factor these special products into linear or irreducible quadratic factors:

1. $x^3 - 27$
2. $x^3 - 3x^2 + 3x - 1$
3. $x^3 + 6x^2 + 12x + 8$
4. $x^4 - 25$
5. $x^4 - 8x^3 + 24x^2 - 32x + 16$

Factor completely by grouping:

1. Example : $\begin{cases} x^3 + 4x^2 - 2x - 8 \\ x^2(x+4) - 2(x+4) \\ \left(x^2 - 2\right)(x+4) \\ \left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)(x+4) \end{cases}$
2. $x^3 + 2x^2 + 3x + 6$
3. $5\cos^2 x - 5\sin^2 x + \sin x + \cos x$
4. $\cos^2 x + 4\cos x + 4 - \tan^2 x$

Use synthetic division to factor as indicated:

1. Example:

$x^3 - 4x^2 + 2X + 1$

| | | | | |
|---|---|---|---|---|
| 1 | 1 | -4 | 2 | 1 |
| | | 1 | -3 | -1 |
| | 1 | -3 | -1 | 0 |

$x^3 - 4x^2 + 2x + 1 = (x-1)(x^2 - 3x + 1)$

2. $2x^3 + 5x + 7 = (x-1)($ $)$
3. $x^4 - 3x^3 + x^2 + x + 2 = (x-2)($ $)$
4. $4x^4 + 3x^3 - 1 = (2x-1)($ $)$

Factor as indicated.

1. $(x-1)^2(x)-(x-1)=(x-1)(\quad)$ Solution: $(x-1)^2(x)-(x-1)=(x-1)\left[(x-1)x-1\right]$
$=(x-1)\left(x^2-x-1\right)$

2. $3\left(x^2+4\right)\left(x^2+1\right)+6\left(x^2+4\right)=3\left(x^2+4\right)(\quad)$

3. $\sqrt{x^2+1}-\dfrac{x^2}{\sqrt{x^2+1}}=\dfrac{1}{\sqrt{x^2+1}}(\quad)$

4. $(x-3)^3(x+2)(x+2)-2(x-3)^2(x+2)^2=(x-3)^2(x+2)(\quad)$

5. $(2x+1)^{3/2}\left(x^{1/2}\right)+(2x+1)^{5/2}\left(x^{-1/2}\right)=(2x+1)^{3/2}\left(x^{-1/2}\right)(\quad)$

6. $(ax+b)^{-1/2}-\dfrac{\sqrt{ax+b}}{b}$ 7. $y^4-(p+q)y^3+\left(p^2q+pq^2\right)y-p^2q^2$

**Reduce each expression to lowest terms and simplify.**

Example: $\dfrac{3x+9}{6x}=\dfrac{3(x+3)}{3(2x)}=\dfrac{x+3}{2x}$ 1. $\dfrac{x^2}{x^{1/2}}$ 2. $\dfrac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

3. $\dfrac{x^{1/2}-x^{1/3}}{x^{1/6}}$ 4. $\dfrac{\sqrt{x-1}+(x-1)^{3/2}}{\sqrt{x-1}}$ 5. $\dfrac{1-(\sin x+\cos x)^2}{2\sin x}$

6. $\dfrac{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}-\frac{1}{4}}$ 7. $1+\dfrac{1}{1-\dfrac{1}{1-\frac{1}{4}}}$

**Solve using the quadratic formula.**

Example: $2x^2+x-3=0$ Solve for x.

$$\begin{cases} 2x^2+x-3=0 \\ x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(1)\pm\sqrt{(1)^2-4(2)(-3)}}{2(2)}=\dfrac{-1\pm\sqrt{1+24}}{4}=\dfrac{-1\pm5}{4} \\ \text{Therefore } x=\dfrac{4}{4}=1 \text{ or } x=\dfrac{-6}{4}=-\dfrac{3}{2} \end{cases}$$

1. $x^2-4x-1=0$ Solve for x. 2. $\cos^2 x+3\cos x+2=0$ Solve for cos x.

3. $x^2-xy-\left(1+y^2\right)=0$ Solve for x. 4. $x^4-4x^2+2=0$ Solve for $x^2$.

**Factor these quadratic expressions.**

1. $x^2 - 3x + 2$
2. $x^2 - 9$
3. $x^2 + 5x - 6$
4. $x^2 + 5x + 6$
5. $2x^2 + 5x - 3$
6. $e^{2x} + 2 + e^{-2x}$
7. $x^4 - 7x^2 + 12$
8. $1 - \sin^2 x$

**Factor these polynomials completely.**

1. $x^3 - 2x^2 - 5x + 6$
2. $2x^3 - 13x^2 + 27x - 18$
3. $9x^4 - 3x^3 + 7x^2 - 3x - 2$
4. $4x^4 - 7x^2 - 5x - 1$
5. $3x^3 - x^2 - 22x + 24$

**Simplify (your life) and these expressions.**

1. $\dfrac{(x-1)(x+3)-(x+1)^2}{(x+1)}$
2. $\dfrac{\sqrt{x^2+1}-\left(\dfrac{1}{\sqrt{x^2+1}}\right)}{x^2+1}$
3. $\dfrac{x^2-5x+6}{x^2-4x+4}$
4. $\dfrac{1}{x+1}-\dfrac{1}{x-1}-\dfrac{1}{x^2-1}$
5. $\dfrac{x(-2x)}{2\sqrt{1-x^2}}+\sqrt{1-x^2}+\dfrac{1}{\sqrt{1-x^2}}$

**Rationalizing: Remove the sum from the denominator by multiplying by the denominator's conjugate.**

1. $\dfrac{1}{1-\cos x}$
2. $\dfrac{x}{1-\sqrt{x^2+1}}$
3. $\dfrac{2}{x+\sqrt{x^2+1}}$

Simplify

1. $\dfrac{x-4}{x^2-3x-4}$

2. $\dfrac{5-x}{x^2-25}$

3. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^3}}$

4. $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

5. $x^{\frac{3}{2}}\left(x+x^{\frac{5}{2}}-x^2\right)$

6. $\dfrac{1}{x+h}-\dfrac{1}{x}$

7. $\dfrac{2}{x^2-6x+9}-\dfrac{1}{x+1}-\dfrac{8}{x^2-2x-3}$

Complete the following identities:

8. $\sin^2 x+\cos^2 x=$ _____

9. $\tan^2 x+1=$ _____

10. $1+\cot^2 x=$ _____

11. $\cos 2x=$ _____

12. $\sin 2x=$ _____

13. $\sin(A+B)=$ _____

14. $\cos(A-B)=$ _____

15. $\tan\dfrac{x}{2}=$ _____

Prove the following identities

16. $\dfrac{\sin x\csc x}{\tan x}=\cot x$

17. $\dfrac{(1+\sin x)^2}{\cos^2 x}=\dfrac{1+\sin x}{1-\sin x}$

18. $\dfrac{\sec A}{\cot A+\tan A}=\sin A$

11. $\cos^4 x-\sin^4 x=\cos^2 x-\sin^2 x$

12. $(1+\sec x)(1-\cos x)=\cos x\tan^2 x$

13. $\sin 2x=\dfrac{2\tan x}{1+\tan^2 x}$

14. $\tan 2B=\dfrac{2\tan B}{\sec^2 B-2\tan^2 B}$

15. $\sin^2\dfrac{x}{2}=\dfrac{\sec x-1}{2\sec x}$

Solve for z:

16. $4x+10yz=0$

17. $y^2+3yz-8z-4x=0$

Rewrite as a single logarithm.

18. $3\ln 2 - \ln 3 + 4\ln 5$

19. $\log_4 3 - \log_4 (x-2) + 3\log_4 (x^3 - 1)$

Simplify

20. $\left(4a^{5/2}\right)^{2/5}$

21. $\left(5a^{2/3}\right)\left(6a^{3/5}\right)$

22. $e^{3\ln 2}$

23. $\ln 1 - 2\ln e^3$

24. $e^{(1+\ln x)}$

25. $\ln e^6$

26. $\log_{1/3} 27$

27. $e^{\ln x/3}$

28. $\dfrac{5xy^{-6}}{15x^{-1/2}y^{-4}}$

28. $\dfrac{2(n+1)!}{6n!}$

Using point-slope form $y - y_1 = m(x - x_1)$, write an equation for the line

29. with slope -3, containing the point (3,4)

30. containing the points (-2,3) and (-5, -6)

31. with slope 0, containing the point (-2, -7)

32. with slope undefined, containing the point (-1, 6)

33. parallel to 3x - 2y = 5 and passes through (-2, 6)

34. perpendicular to the line in #29, containing the point (3, 4)

35. Find the amplitude, period, vertical shift, and phase shift for $y = \frac{3}{4}\sin(3x - \pi) + 2$. Be sure to label your answers carefully.

36. Find the remainder for the division of $x^7 - 5x^5 + 6x^3 - 2x + 16$ by $x + 1$.

Without a calculator, determine the **exact** value of the following expressions:

37. $\sin 0$ 38. $\sin\frac{\pi}{2}$ 39. $\sin\frac{3\pi}{4}$ 40. $\cos\pi$ 41. $\cos\frac{7\pi}{6}$

42. $\cos\frac{\pi}{3}$ 43. $\tan\frac{7\pi}{4}$ 44. $\tan\frac{\pi}{3}$ 45. $\tan^{-1}1$ 46. $\tan\frac{\pi}{2}$

47. $\cos\left(\sin^{-1}\frac{1}{2}\right)$ 48. $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

For each function, determine its domain and range.

| | Function | Domain | Range |
|---|---|---|---|
| 49. | $y=\sqrt{x-4}$ | _____ | _____ |
| 50. | $y=\sqrt{x^2-4}$ | _____ | _____ |
| 51. | $y=\sqrt{4-x}$ | _____ | _____ |
| 52. | $y=\sqrt{x^2+4}$ | _____ | _____ |

If $f(x)=\sqrt[3]{x-3}$ and $g(x)=x^2+2$, determine each of the following (Recall that $(f\circ g)(x)=f(g(x))$)

53. $(f\circ g)(-2)$ 54. $(g\circ f)(-2)$

55. $(f\circ g)(x)$ 56. $f^{-1}(x)$

Expand and simplify.

57. $\sum_{n=0}^{4}\frac{n^3}{2}$ 58. $\sum_{n=1}^{3}\frac{1}{n^2}$

Solve the following inequalities and graph the solutions on a number line.

59. $5<2x+7<12$ 60. $1-3x\le 10$

61. $|2x+5|<7$ 62. $|3x-4|>|2x+1|$

Solve for x, where x is a real number. Show the work that leads to your solution.

63. $|5x-2|=8$

64. $x^2+3x-4=14$

65. $e^{4x}=3$

66. $\log x+\log(x-3)=1$

67. $3\sin^2 x=\cos^2 x,\ 0\le x<2\pi$

68. $\sin 2x=\sin 2,\ 0\le x<2\pi$

Determine all points of intersection:

69. parabola $y=x^2+3x-4$ and line $y=5x+11$

70. $y=\cos x$ and $y=\sin x$ in the first quadrant

71. Find the amplitude, period, vertical shift, phase shift and increments for $y=3\sin(4x-\pi)+2$. Be sure to label your answers.

Graph the following without the use of a graphing calculator. Be sure to include all intercepts, asymptotes, holes, increments, domain, range, and the appropriate number of points to clearly distinguish from one graph to the next.

72. Sketch the graph from problem # 71.

These graphs are called "parent graphs." They are to be committed to memory.

73. The square function $y=x^2$

74. The cubic function $y=x^3$

75. The square root function $y=\sqrt{x}$

76. The absolute value function $y=|x|$

77. The identity function $y=x$

78. The cube root function $y=\sqrt[3]{x}$

79. The natural log function $y=\ln x$

80. The exponential function $y=e^x$

81. The sine function $y = \sin x$

82. The cosine function $y = \cos x$

83. The tangent function $y = \tan x$

84. The inverse function $y = \frac{1}{x}$

85. The inverse squared function $y = \frac{1}{x^2}$

# Name _____ Calculus Summer Packet

**Tell whether the following relationships are functions**.

1) {(v, -1), (u, 2), (w, 0), (u, -2)}

2) {(6, 2), (12, 2), (14, 2)}

3) $f(x) = x^3 + x^2 - 5$

4) $y^2 + x^2 = 1$

**Given the slope and a point on the line, give the equation of the line.**

5) (2,1) m=0

6) (7, -2) m=1/2

7) (-4,1) m=undefined

8) (0,9) m=-2

**Determine algebraically whether the lines $L_1$ and $L_2$ are parallel, perpendicular, or neither.**

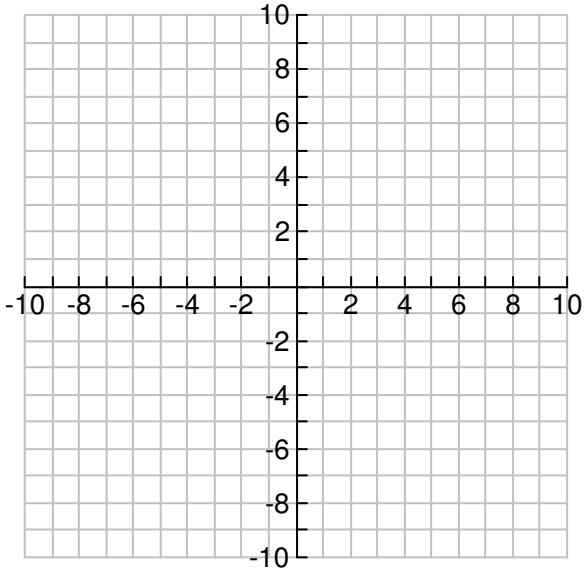
9) $L_1$: (0,-1), (5,9)
$L_2$: (0,3), (4,1)

10) $L_1$: (-2,1), (1,5)
$L_2$: (1,3), (5, -5)

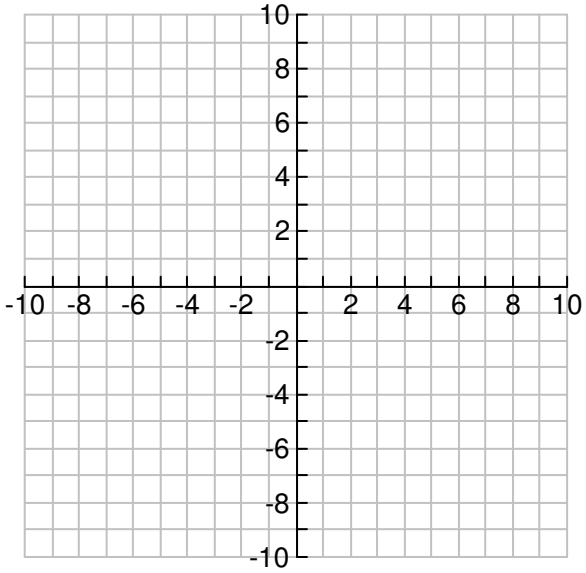
11) $L_1$: (3,6), (-6,0)
$L_2$: (0,-1), (5, 7/3)

**Use a graphing calculator to graph the following functions; state the domain, range, intercepts, and asymptotes (if they exist).**

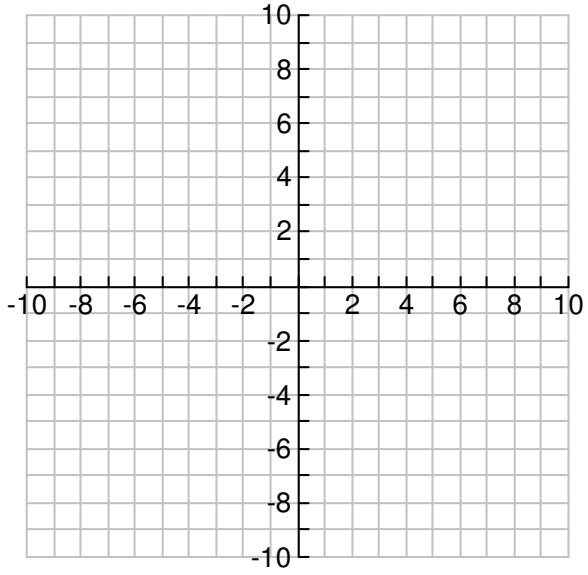
12) $f(x) = (x-2)^3 + 4$



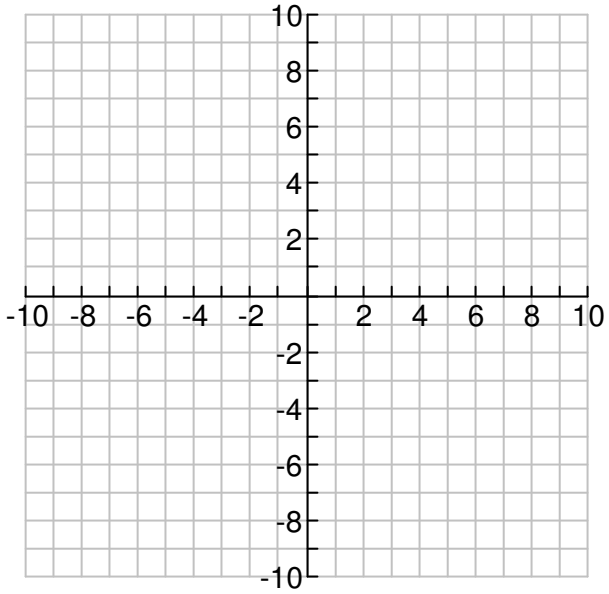

13) $f(x) = \sqrt{-x+4} - 2$



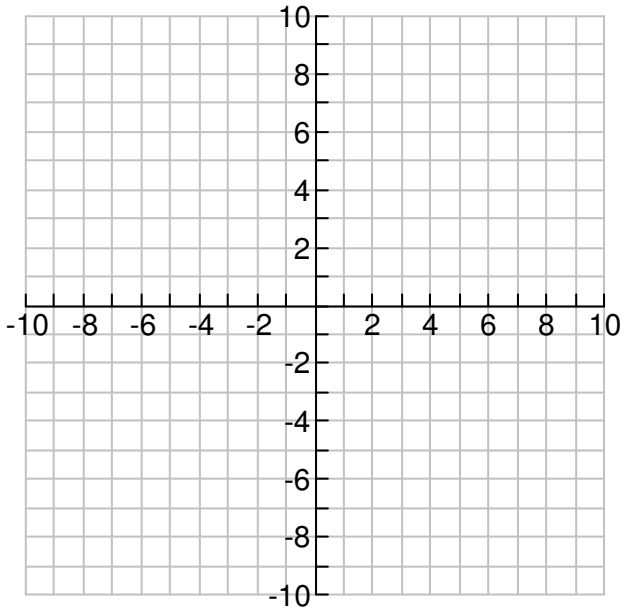
14) $f(x) = |x+2| - 3$



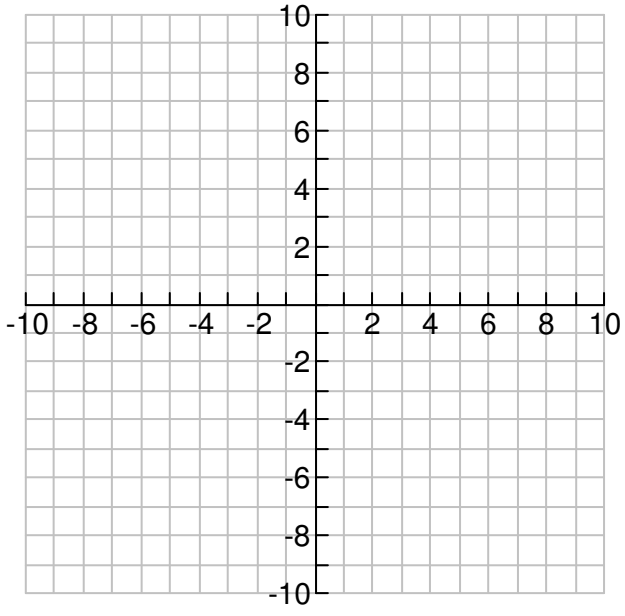

15) $f(x) = \dfrac{3}{x \ -2}$



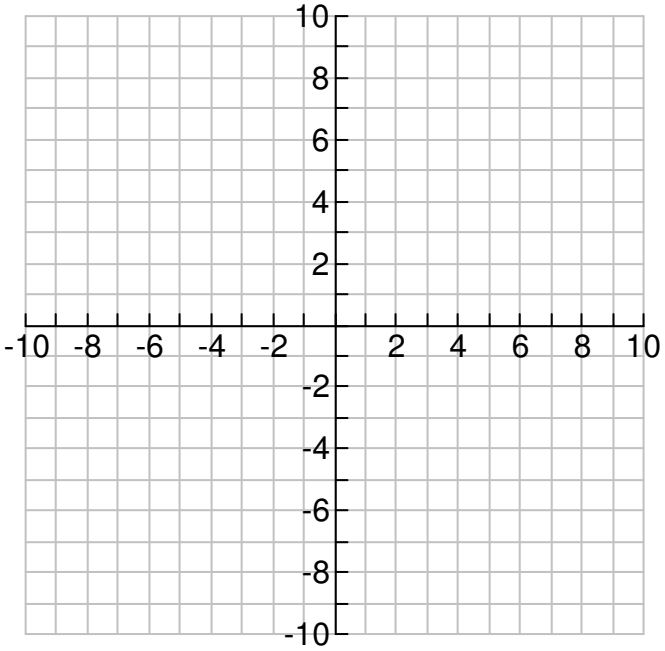
16) $f(x)=\dfrac{x+2}{x^2-5x-6}$



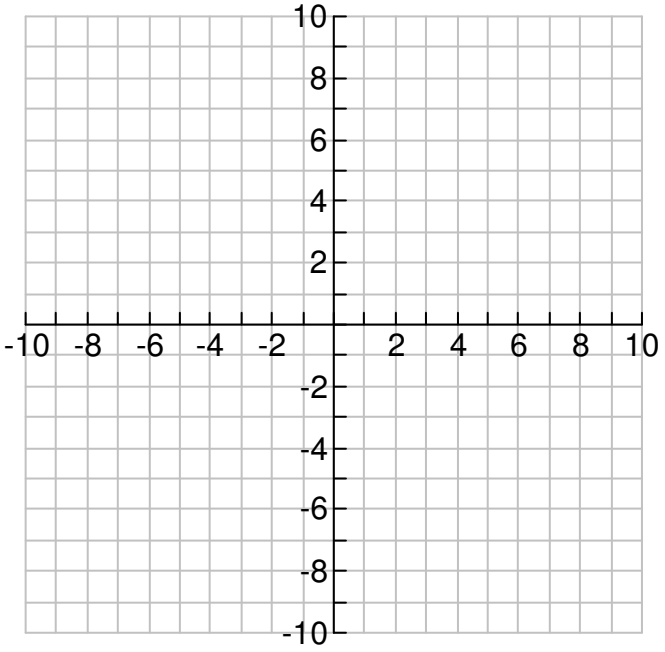

17) $f(x)=\begin{cases} x^2+2, x\le 1 \\ 2x^2+2, x>1 \end{cases}$



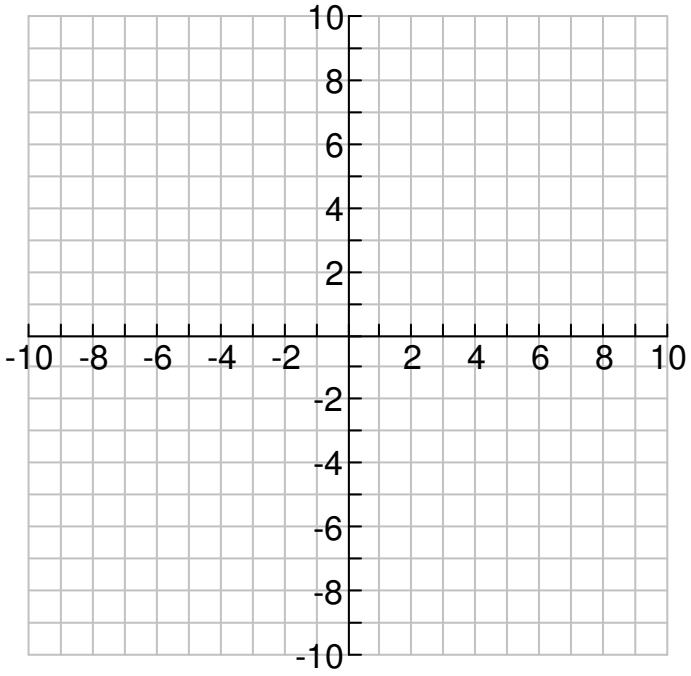
18) $f(x) = 3\sin(x - \frac{\pi}{4})$




19) $f(x) = \tan(2x - \frac{\pi}{3})$



20) $f(x) = -\csc 4x$




**Show how you would algebraically find the Domain and Range for each function.**

21) $f(x) = \dfrac{5}{x^2 - 2x}$ Domain: _____ Range: _____

22) $f(x) = \dfrac{1}{\sqrt{x^2 + 8x}}$ Domain: _____ Range: _____

23) $f(x) = \dfrac{3x}{x^2 + 1}$ Domain: _____ Range: _____

24) $f(x) = \sqrt[7]{15x^3 - 3}$ Domain: _____ Range: _____

25) f(x) = $\dfrac{1}{\sqrt{2 + 5x}}$ Domain: _____ Range: _____

26) $f(x) = -2x^2 + 3$ Domain: _____ Range: _____

27) $f(x) = |x + 3|$ Domain: _____ Range: _____

28) $f(x) = \begin{cases} x^2 + 2, x \le 1 \\ 2x^2 + 2, x > 1 \end{cases}$ Domain: _____ Range: _____

29) $f(x) = \sin x$ Domain: _____ Range: _____

30) $f(x) = \sec x$ Domain: _____ Range: _____

31) $f(x) = \tan x$ Domain: _____ Range: _____

32) $f(x) = 3\cos(x - \frac{\pi}{2})$ Domain: _____ Range: _____

33) $f(x) = -2\csc x$ Domain: _____ Range: _____

34) $f(x) = 2 + \cot x$ Domain: _____ Range: _____

35) Given $f(x) = 6 - 2x^2$, find f(-3). Domain: _____ Range: _____

**For # 36 and 37 use algebra to find the value(s) of *x* for which $f(x) = g(x)$.**

36) $f(x) = 2x^2 - 6x$
$g(x) = x - 3$

37) $-2 = 2x - 3y$
$6 = 4x - y$

38) If $f(x) = x^2 - 3x$, find: $\frac{f(x+h) - f(x)}{h}$

39) Is $f(x) = \frac{3}{x}$ one-to-one? (Does it have an inverse?)
If it is, find the inverse function.

40) Is $f(x) = x^4$ one-to-one? (Does it have an inverse?)

If it is, find the inverse function.

41) Determine whether the functions $f(x) = \sqrt{x^2 - 5}$
and $g(x) = x^2 + 5$ are inverses of each other.

42) Determine whether the functions $f(x) = (x+1)^3$ and
$g(x) = \sqrt[3]{x} - 1$ are inverses of each other.

***Find <u>all</u> solutions for the given polynomial equation.*** *You will need to use at least one of the methods listed below, in some cases more than one may be needed. You may use a graphing calculator, but answers should be exact answers, not decimal approximations.*

(1) factoring
(2) quadratic formula
(3) synthetic division

43) $f(x) = x^2 + 3x - 4$ _____

44) $f(x) = 16x^2 + 30x + 100$ _____

45) $f(x) = 2x^3 + 6x^2 + 10x$ _____

46) $f(x) = x^3 + x^2 - 5x + 3$ _____

47) $f(x) = x^3 + 64$ _____

48) $f(x) = x^4 + 4x^3 - 7x^2 - 22x + 24$ _____

49) $f(x) = x^5 + 3x^4 + 5x^3 + 7x^2 + 9x + 5$ _____

**Verify the following trig identities**.

50) $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\csc\theta$ _____

51) $1+\tan^2 x=(\sin x+2)(\sin x-1)$ _____

52) $\sin^2 x+\sin x-2=\dfrac{\sec^2 x-1}{\sec^2 x}$ _____

53) $-\cos^2 x+\sin x-1=\dfrac{1}{\csc^2 x}$ _____

**Use the fundamental identities to simplify the expression.**

54) $\sin\phi(\csc\phi-\sin\phi)=$ _____

55) $\sec^2 x(1-\sin^2 x)=$ _____

56) $\dfrac{\csc\theta}{\sec\theta}=$ _____

.

57) $\dfrac{1}{\tan^2 x+1}=$ _____

58) $\dfrac{\tan^2\theta}{\sec^2\theta}=$ _____

59) $\cot\left(\dfrac{\pi}{2}-x\right)\cos x=$ _____

60) $\cos t(1+\tan^2 t)=$ _____

**Perform the multiplication and use the fundamental identities to simplify.**

61) $(\sin x + \cos x)^2$ _____

62) $(\cot x + \csc x)(\cot x - \csc x)$ _____

63) $(\sec x + 1)(\sec x - 1)$ _____

64) $(3 - 3\sin x)(3 + 3\sin x)$ _____

**Perform the addition or subtraction and use the fundamental identities to simplify**.

65) $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x}$ _____

66) $\dfrac{1}{\sec x + 1} + \dfrac{1}{\sec x - 1}$ _____

67) $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x}$ _____

68) $\tan x - \dfrac{\sec^2 x}{\tan x}$ _____

**Rewrite the expression so that it is not in fractional form.**

69) $\dfrac{\sin^2 y}{1-\cos y}$ _____

70) $\dfrac{5}{\tan x + \sec x}$ _____

71) $\dfrac{3}{\sec x - \tan x}$ _____

72) $\dfrac{\tan^2 x}{\csc x + 1}$ _____